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COMMENT

Reply to 'Critical dynamics of the spin-exchange model in quasilinear fractal geometries'

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Abstract. As pointed out in the preceding comment, the claim, made by us, that $z=3$ for the one-dimensional Ising model with conserved order parameter is erroneous. The authors show that the origin of the error lies in the fact that this model is not capable of relaxing to equilibrium at the critical temperature $T=0$.

As pointed out in the preceding comment (Luscombe 1987), our original comments on the derivation of the dynamic exponent for one-dimensional systems contains an erroneous treatment of the spin-exchange Ising model. We wish to show here how the error arose, as well as the possible significance of the exponent we obtained.

As mentioned in our original comment (Leyvraz and Jan 1986), the calculation we performed was an RG following the ideas of Jan *et al* (1983) and Kalle (1984). This meant starting with an arbitrary initial state (typically a completely disordered state) and using the power law describing the growth of the domain length as a function of time to infer the value of the exponent $1/z$. This is justified by the fact that, at T_c, a renormalisation of length by a factor b entails a renormalisation of time by a factor b^z. Thus it can be inferred that the time necessary for a domain to increase by a factor of b scales with b as b^z. From this it then follows that $L(t) \sim t^{1/z}$, if $L(t)$ denotes the average domain length of time t. This is, however, only correct as the system approaches equilibrium, i.e. in general, as time goes to infinity. As we shall see, a neglect of this point is the source of the error involved in the spin-exchange Ising model case.

Let us now consider the case of Glauber dynamics for the one-dimensional Ising (or Potts) model. In that case, the only transition probabilities that are non-singular at $T=0$ are the following:

$$\begin{aligned} p(s_i \to s_i') &= 0 \qquad \text{if } \Delta E > 0 \\ &= 1 \qquad \text{if } \Delta E < 0 \\ &= \alpha \qquad \text{if } \Delta E = 0 \end{aligned} \tag{1}$$

where s_i denotes the value of the spin at position i, ΔE denotes the energy change occurring upon changing s_i to s_i' and α is an arbitrary number strictly greater than zero and less than or equal to one. Under these circumstances, the domain walls perform uncorrelated random walks and the unit of time is such that a wall moves on the average by one lattice spacing in α^{-1} time units. Two walls then always either annihilate or stick irreversibly to one another upon contact. This is, however, a well studied reaction-diffusion system, where the particle density is known to decay as $t^{-1/2}$ (Toussaint and Wilczek 1983, Torney and McConnell 1983a, b). Thus the result $z = 2$ is obtained in a rather general manner for these reaction rates independently of q. Incidentally, the one-dimensional 'solid-on-solid' model is the one case we have found with a dynamic exponent z different from two ($z = 4$). The reason for this lies in the existence of two types of walls such that one type will, in general, only annihilate with the other, thus leading (Toussaint and Wilczek 1983) to the density of walls decaying as $t^{-1/4}$. Note, however, that the choice of rates suggested by Lage (1985) leading to $z = 3$ is in fact singular at $T = 0$, so that the system cannot relax to its ground state in finite time, since the time necessary for a wall to move is of the order of $\exp(J/k_B T)$. Thus the reasoning outlined above does not apply. This is all in good agreement with the results of Weir and Kosterlitz (1986), Weir *et al* (1986) as well as Cordery *et al* (1981).

This brings us to the heart of the problem in the case of the spin-exchange kinetic Ising model: in this case it is *never* possible to relax to the ground state at $T = 0$, as any cluster of length greater than two is energetically stable under spin exchange. The obvious way to avoid this problem is to consider the relaxation of the system, at a temperature so low that any diffusion process can be considered instantaneous compared to the timescale at which a particle is separated from a cluster. This, however, implies $L(t) \ll \xi(T)$, where $\xi(T)$ is the coherence length of the system at the infinitesimal temperature T at which it is maintained. This means that the exponent we derive is typical of the system far from its equilibrium state, so that the power law $L(t) \sim t^{1/3}$ derived for this regime cannot be used to draw any inferences concerning the exponent z. From these remarks then follows the fact that the renormalisation group approach suggested by Jan *et al* fails whenever the dynamics are such that the system cannot relax to equilibrium at the critical temperature. This is, of course, only possible if the critical temperature is zero.

Let us now make a final remark concerning the numerical tests we made. These were always tests of the specific reaction-diffusion model under consideration, not independent evaluations of the dynamic exponent z. Thus, for the spin-exchange kinetic Ising model, we looked at the growth of domains where the unit of time was taken to be the time necessary to dissociate a particle from the cluster to which it belonged and the time for diffusion was identically zero. Under those circumstances, the growth of $\frac{1}{3}$ was indeed verified to good accuracy. These simulations are quite easy to duplicate and we do no wish to overburden a short note by reporting these numerical tests.

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